Model Predictive Control Under Uncertainty: Theory, Computations and Applications

Saša V. Raković, William S. Levine, Behçet Açıkmeşe and Ilya V. Kolmanovsky

Abstract— **This workshop introduces its audience to the theory, design and applications of model predictive control (MPC) under uncertainty. The workshop provides conceptual and technical principles governing rigorous and computationally effective methods for design of MPC under set–membership and probabilistic uncertainty. The theoretical fundamentals are carefully introduced and studied within the frameworks of robust MPC and stochastic MPC. The technical foundations are complemented with a discussion of related design and practical aspects as well as with an overview of effective computations based on convex and reliable real–time optimization. Thus, the workshop provides a unique and unified exposure to key constituents of MPC under uncertainty. It also allows for a comprehensive understanding of highly powerful and applicable robust and stochastic MPC algorithms that have seen real–life transition from theory to practice in several important applications within aerospace and automotive industries. Finally, a carefully designed panel discussion at the workshop assesses the current state of affairs in, and identifies relevant future research directions for, MPC under uncertainty.**

I. PRÉCIS

A. Essentials

Organizers

The organizers of the workshop are:

1.1. Saša V. Raković,
University of Texas, Austin, USA.
1.2. William S. Levine,
University of Maryland, College Park, USA.
1.3. Behçet Açıkmeşe,
University of Texas, Austin, USA (until 12/2015),
University of Washington, Seattle, USA (from 01/2016).
1.4. Ilya V. Kolmanovsky,
University of Michigan, Ann Arbor, USA.

Brief Introduction

MPC [1] is a modern, dynamic and cross–disciplinary field based on mathematical control theory that fuses (and synergistically treats) control–theoretic issues (e.g. stability, performance, robustness, etc.) for constrained systems with the optimization theory [2]–[5] and numerical computations [6], [7]. Being a fundamental and contemporary research area, MPC has seen advances addressing a broad range of underlying theoretical and implementation issues. MPC's role in control engineering practice is best evidenced by the large number and versatility of its real life implementations across many domains of industry and technology [8]. The interest in MPC is rapidly growing due to its inherent ability to simultaneously handle constraints, guarantee stability and optimize performance in a systematic way. For a nonexhaustive overview of this important field see, for instance, monographs [1], [9]–[15] and survey papers [16], [17].

The extensive research over the last few decades has resulted in fairly well developed theory of MPC and a clear perspective on still open research problems. Important MPC subfields in which significant progress has recently been made include robust and stochastic MPC (i.e. MPC accounting for set–membership or probabilistic representation/description of the uncertainties) as well as efficient computations for MPC via convex and reliable real–time optimization. Thus, a workshop on MPC under uncertainty via reliable real–time optimization will serve two fundamental and useful purposes. Firstly, it will acquaint its audience with the basic principles and latest developments in MPC under uncertainty and its real–time implementation. Secondly, it will focus the attention on the areas of great practical importance in control applications that have already initiated, and will continue spanning, many interesting directions for future research (e.g., MPC for classes of large–scale, decentralized, distributed, autonomous, cyber–physical and networked systems as well as secure and resilient MPC.)

B. Primary Objectives

The major goals of the workshop are to provide a comprehensive tutorial on both fundamental and advanced aspects of MPC under uncertainty and present a unified treatment of its conceptual and practical aspects. More specifically, the workshop will deliver a compact understanding of MPC as well as comprehensive theoretical foundation underpinning MPC under both the set–membership and stochastic uncertainty (a.k.a. robust and stochastic MPC). The workshop will also cover the numerical implementation of conventional, robust and stochastic MPC design methods via advanced and reliable real–time optimization techniques. Thus, the workshop will synergistically fuse theoretical, computational and applications–driven aspects of MPC under uncertainty and, consequently, furnish a unique blend of advanced control synthesis and analysis methods.

An equally important aim of the workshop is to highlight the importance of MPC under uncertainty and disseminate the knowledge of advanced robust and stochastic MPC as powerful design methods with high potential to successfully tackle real–life problems across a wide range of traditional and emerging industrial applications.

Saša V. Raković and Behçet Açıkmeşe are with the University of Texas at Austin, USA. Behçet Açıkmeşe will be with the University of Washington, Seattle, USA as of January 01, 2016. William S. Levine is with the University of Maryland at College Park, USA. Ilya V. Kolmanovsky is with the University of Michigan at Ann Arbor, USA.

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The workshop will also provide, via a carefully designed panel discussion, an assessment of the current state of MPC under uncertainty, and it will highlight future research directions for this field in terms of theory and applications. Finally, the workshop will stimulate the creation of a specialized network of researchers focused on further advances in this highly important research field.

C. Format

Style

The style of the workshop is tutorial and pedagogical. An optimally designed concoction of conceptual, implementation and technical aspects is delivered in an easy–to–follow step–by–step format. The necessary fundamental concepts and mathematical techniques of MPC and MPC under uncertainty are introduced first. This base is built upon by providing a concise overview of the theory underpinning MPC under set–membership and probabilistic uncertainty. The theory of robust and stochastic MPC is complemented with discussion of related algorithmic and practical aspects, and its transfer to practice is demonstrated by furnishing some important real–life applications from aerospace and automotive industries.

Intended Audience

The workshop is designed carefully and flexibly in order to be accessible to a broad range of researchers and engineers within both academia and industry. On one hand, the workshop provides, for junior researchers and students, a comprehensive exposure to advanced theory and design of MPC for constrained systems subject to uncertainty. On the other hand, the workshop delivers a systematic framework for senior researchers and engineers working on real–life industrial problems where constraints and uncertainty play a key role.

Prerequisites

The graduate level courses in systems and control theory, linear algebra and foundation of optimization provide an ideal background for the workshop. Since the workshop topics are sufficiently self–contained, the workshop can, in principle, also be attended by less specialized audience.

Handouts

The workshop handouts include presenters slides and notes, fact sheets and lists of reference for further reading.

Organization

Length: The workshop is planned as one full day event.

Topics: The workshop focuses uniformly on the following main research themes.

2.1. Model Predictive Control.
2.2. Robust Model Predictive Control.
2.3. Stochastic Model Predictive Control
2.4. Convexification for Model Predictive Control under uncertainty with reliable online computations

The workshop also provides real–life applications and reports on the actual transition from theory to practice.

Presenters: The workshop will be entirely presented by the four organizers in a coherent and effective manner. The panel discussion at the end of the workshop will include additional participants.

Synergy: The workshop is synchronized, and designed to be compatible, with a related workshop proposed for ACC–2016 and that is entitled "Model Predictive Control" and organized by James B. Rawlings and Thomas A. Badgwell. Compared to the latter, our workshop addresses more advanced techniques and topics that deal with MPC under uncertainty. In addition, this workshop complements and provides background for a double invited session entitled "MPC, Quo Vadis?" which focuses on advanced topics in MPC and is also organized by us for the ACC–2016.

II. SYNOPSIS

A. Content

3.1. MPC. (Speaker: William S. Levine)
- Conventional MPC.
    - Paradigm and formulation.
    - Algorithmic implementation.
    - Fundamental properties.
- Uncertainty in MPC.
    - Types and models.
    - Interplay with predictions, constraints and cost.
    - Inherent robustness and direct consideration.

3.2. Robust MPC. (Speaker: Saša V. Raković)
- Robust MPC I.
    - Exact robust MPC.
    - Parameterized predictions for robust MPC.
    - Tube MPC.
- Robust MPC II.
    - Rigid tube MPC. (Formulation, implementation and properties.)
    - Homothetic tube MPC. (Formulation, implementation and properties.)
    - Parameterized tube MPC. (Formulation, implementation and properties.)

3.3. Stochastic MPC. (Speaker: Ilya V. Kolmanovsky)
- Stochastic MPC I.
    - Modelling and representation of stochastic uncertainties.
    - Specification and treatment of probabilistic constraints and average cost.
    - Formulation and computational treatment of stochastic MPC problems for linear systems.
- Stochastic MPC II.
    - Formulation and computational treatment of stochastic MPC problems for nonlinear systems.
    - Guaranteeing recursive feasibility and stability in stochastic MPC problems.

  - ○ Treatment of special problems such as dual control and drift counteraction in a receding horizon framework.

3.4. Convexification for MPC under uncertainty with reliable online computations. (Speaker: Behçet Açıkmeşe)
- Convex optimization in MPC.
  - ○ Convex optimization.
  - ○ Convexification techniques in optimal control.
  - ○ Convex optimization for MPC under uncertainty.
- Reliable real–time optimization for MPC.
  - ○ Real–time optimization.
  - ○ Custom interior point methods algorithms.
  - ○ Reliable real–time optimization solvers.

3.5. Overview of Applications.
(Speakers: Ilya V. Kolmanovsky and Behçet Açıkmeşe)
- Applications in automotive industry.
  - ○ Robust Model Predictive Control of diesel engines.
  - ○ Stochastic Model Predictive Control of vehicle cruise management.
  - ○ Stochastic Model Predictive Control of vehicle energy management.
- Applications in aerospace industry.
  - ○ Planetary soft landing.
  - ○ Formation flying with ON/OFF actuators.
  - ○ Motion planning for quadrotors.

MPC

Conventional MPC

MPC optimizes predicted system behaviour in order to determine the best current control action. An optimal forecast of the system behaviour is induced from open–loop optimal control that is cast as an online optimization and that takes account of system dynamics, constraints and control objectives. The online optimization, solved at the current time instant and state of the system, yields an optimal sequence of open–loop predicted control actions, the first element of which is selected as the best current control action. This procedure is continually applied throughout the control process.

This part of the workshop introduces MPC paradigm, provides its formulation, discusses its algorithmic implementation and summarizes its fundamental system theoretic properties.

Uncertainty in MPC

When the model of the system is exact, constraints perfectly capture underlying limitations and restrictions and no uncertainty is present, the open loop and closed loop control strategies yield the same outcome. Unfortunately, perfect exactness is seldom available and many unaccounted for discrepancies affect the actual control process. The frequent sources of uncertainty include model errors, unknown disturbances and state information imperfections. Thus, it is crucial to take the uncertainty and its effect in MPC into account either indirectly via *a–posteriori* analysis or directly via *a–priori* synthesis.

This part of the workshop discusses primarily types and models for the uncertainty in MPC as well as interplay of the uncertainty with predictions, constraints and cost. Furthermore, it also comments on inherent robustness of MPC and provides a base for design methods that take the uncertainty into account more directly, i.e. robust and stochastic MPC.

Robust MPC

Robust MPC I

Robust MPC is an improved MPC variant that ensures robustness against the set–membership uncertainty. The exact robust MPC can be seen as a repetitive decision making process in which, the basic decision making reduces to a convoluted closed–loop robust optimal control. In fact, the exact robust MPC requires online infinite-dimensional minimaximization over the sequence of control laws in a stark contrast to a finite–dimensional minimization over the sequence of control actions in the conventional case. A dominant problem stems from the facts that the exact robust MPC provides strong structural properties but it is computationally unwieldy, while the conventional MPC is not necessarily robust even though it is computationally convenient. The key challenge is to devise robust MPC methodology that employs a generalized prediction framework and allows for a meaningful and practicable optimization of, and over, the set of possible system behaviours effected by the uncertainty.

This part of the workshop focuses on exact robust MPC and, motivated by its computational intractability, it highlights importance of careful use of parametrized control policies in order to computationally simplify the exact robust MPC whilst preserving as many of its strong structural properties as possible. A particular emphasis is given to tube MPC framework that addresses effectively the fundamental challenge of reaching a meaningful compromise between the quality of guaranteed structural properties and the associated computational complexity.

Robust MPC II

Tube MPC considers predicted behaviour in terms of the sets of possible states and controls due to the spread of trajectories caused by the uncertainty. The related state and control tubes represent either the exact or outer bounding sequences of the sets of possible states and associated controls. The sensible parameterizations of the state and control tubes and the associated tubes control policy lead to computationally highly attractive tube MPC that induces strong structural properties. Tube MPC has proven particularly effective for constrained linear systems subject to additive uncertainty.

This part of the workshop builds upon generic introduction to tube MPC by providing an overview (in terms of theory and design aspects for) of basic and advanced tube MPC design methods, namely rigid, homothetic and parameterized tube MPC synthesis.

Stochastic MPC

In this part of the workshop, variants of MPC frequently referred to as stochastic MPC (SMPC) will be described. The SMPC is distinguished by the ability to explicitly consider and address probabilistic specifications of the uncertainties. From practical standpoint, SMPC has a potential to reduce the conservatism by incorporating the stochastic information about the uncertainties and through probabilistic constraint handling. In many aspects, the use of SMPC resembles that of stochastic optimal control and dynamic programming, however, emphasizing constraint handling. The presentation of stochastic MPC will consist of two parts.

Stochastic MPC I

The presentation of SMPC will begin with a more in-depth discussion of modeling/representation of stochastic uncertainties and probabilistic handling of constraints, and it will be demonstrated that these issues have to be carefully considered. For instance, naive formulations of SMPC problems may lead to constraint violation by every trajectory with probability 1. Then, existing approaches to SMPC for linear systems will be considered. In particular, scenario–based approaches to predicting the evolution of uncertain parameters which in cases when the parameter–dependent model is linear and the cost is quadratic, reduce the MPC problem to a sparse quadratic programming problem will be discussed. Other approaches to treating SMPC problems for linear systems with additive probabilistic uncertainties will be covered as well.

Stochastic MPC II

The developments will then be extended to the case of SMPC for general nonlinear systems. Mechanisms for guaranteeing recursive feasibility and stochastic stability properties in SMPC problems will be described. Finally, special problems such as of dual (i.e., combined identification/estimation and tracking) control and drift counteraction/optimal stopping will be introduced, and their treatment within receding horizon framework will be covered as motivated by various practical applications.

Convexification for MPC under uncertainty with reliable online computations

This part of the workshop will present methods of convexification and real–time convex optimization for robust and stochastic MPC problems.

Convex optimization in MPC

Optimization has traditionally been regarded as unsuitable for onboard real–time use in many fields. The main reason for this was a hard requirement for *a–priori* guarantees that a solution to an optimization problem can be computed in an autonomous manner and in real–time while relying on limited onboard processing capabilities. While satisfying such a condition for general optimization problems may not be possible, convex optimization problems can be solved quickly to global optimality in a predetermined amount of computations. In other words, one can provide guarantees for finding an optimal solution without a human in the loop.

This part of the workshop introduces recent analytical convexification results enabling the formulation of a class of MPC problems within convex optimization framework. It also presents LMI based methods for handling deterministic and probabilistic disturbances in MPC with particular emphasis on the model uncertainties described via incremental quadratic inequalities.

Reliable real–time optimization for MPC

Real–time optimization is an essential component of MPC, especially when it comes to MPC's implementation to contemporary applications in aerospace, autonomous vehicles and smart grid. Recent significant advances in the design of reliable real–time solvers for Linear Programming (LP), Quadratic Programming (QP), and Second Order Cone Programming (SOCP) have opened opportunities for the improvement of existing MPC techniques and for the development of novel techniques that exploit theoretically sound MPC under uncertainty.

This part of the workshop summarizes recent advances in real–time optimization and convex optimization. A particular emphasis is placed on the development of custom Interior Point Method (IPM) algorithms and methods for customization and autocoding that lead to real–time implementable software.

Overview of Applications

Applications of MPC under uncertainty have been reported in many domains, including finance, building control, electric power grid, chemical process industry, and automotive and aerospace systems. The applications to automotive and aerospace systems are of special interest as these systems operate in uncertain environment, have fast dynamics and very limited onboard computing power. Consequently, they will be the focus of the workshop.

Applications in automotive industry.

After providing a brief survey and a summary of various applications of robust and stochastic MPC for automotive systems that were reported in the published literature, several examples will be considered in more detail.

The first example involves robust MPC for diesel engine air path control. In this engine control problem, an Exhaust Gas Recirculation (EGR) valve and a Variable Geometry Turbocharger are coordinated to enable tracking of intake manifold pressure and EGR rate set–points subject to pointwise–in–time constraints on actuator ranges, intake manifold pressure, exhaust manifold pressure and turbocharger speed. The set–bounded additive uncertainties are handled using a tube MPC approach. Experimental results are reported.

The second and third examples will illustrate the applications of stochastic MPC to the vehicle adaptive cruise control and Hybrid Electric Vehicle (HEV) energy management. In the vehicle adaptive cruise control problem, the control of an automotive vehicle that follows a lead vehicle the acceleration of which is probabilistically modeled and constraints on acceleration, velocity and relative distance need be enforced will be treated. In the energy management problem for a battery Hybrid Electric Vehicle a probabilistic representation for the driver wheel power demand will be employed and constraints on the battery state of charge will be enforced.

Applications in aerospace industry

The first aerospace application is planetary soft landing. Planetary soft landing is one of the benchmark problems of optimal control theory and has gained significant interest due to Mars exploration and reusable launch vehicles. The soft landing problem with all relevant constraints can be posed as a finite–horizon optimal control problem with state and control constraints [18]. The online, real–time, generation of fuel–optimal trajectories to a prescribed location on a planet's surface is a challenging problem due to the constraints on the fuel, the control inputs, and the states. The main difficulty in solving this constrained problem is the existence of nonconvex constraints on the control input, which are due to a nonzero lower bound on the control input magnitude and a nonconvex constraint on its direction. We introduced new convexification methods for the control constraints that is proven to be lossless; i.e., an optimal solution of the soft landing problem can be obtained via solution of the proposed convex relaxation of the problem [19]. The lossless convexification enables the use of interior point methods of convex optimization to obtain optimal solutions of the original nonconvex optimal control problem. We also developed real-time Interior Point Method (IPM) algorithms to compute optimal trajectories in flight, which is demonstrated by NASA JPL in recent flight experiments (see Figure 1).


Fig. 1. A rocket test by NASA in 2013.

The second example is rendezvous of any number of spacecraft using only the relative aerodynamic drag forces between the vehicles. Each spacecraft is equipped with drag plates that can be opened or closed, thus modulating the drag force acting on the vehicle, i.e., ON/OFF control. The optimal control problem is relaxed to a convex problem, and it is proved that a solution of the relaxed problem exists that is also a solution of the original problem. This process leads to a lossless convexification, which then leads to a solution method based on solving a finite number of linear programming problems [20]. Two examples are solved with two and five vehicles and the results are compared with existing technologies. Because the method is fast and globally convergent, it is well suited for real–time use onboard a vehicle.

The third application considers optimal control problems for quadrotors, which are increasingly being used in many applications. The problem has a nonconvex control constraint. By using the general convexification techniques we developed, this problem is also convexified so that the real–time IPM methods can be used to obtain online solutions.

B. Schedule

The anticipated schedule is as follows.

08:30 – 08:40 Introductions
(William S. Levine)
08:40 – 09:20 Conventional MPC
(William S. Levine)
09:20 – 10:00 Uncertainty in MPC
(William S. Levine)

10:00 – 10:30 Break

10:30 – 11:15 Robust MPC I
(Saša V. Raković)
11:15 – 12:00 Robust MPC II
(Saša V. Raković)

12:00 – 13:00 Lunch

13:00 – 13:45 Stochastic MPC I
(Ilya V. Kolmanovsky)
13:45 – 14:30 Stochastic MPC II
(Ilya V. Kolmanovsky)

14:30 – 14:45 Break

14:45 – 15:30 Convex optimization in MPC
(Behçet Açıkmeşe)
15:30 – 16:15 Reliable real–time optimization for MPC
(Behçet Açıkmeşe)

16:15 – 16:30 Break

16:30 – 17:15 Overview of Applications
(Ilya V. Kolmanovsky and Behçet Açıkmeşe)

17:15 – 18:00 Closing Panel Discussion
(William S. Levine and Saša V. Raković)

III. PRESENTERS

A. Saša V. Raković

Dr. Saša V. Raković received the M.Sc. degree in Control Engineering and Ph.D. degree in Control Theory from Imperial College London in 2001 and 2005, respectively. His Ph.D. thesis, entitled "Robust Control of Constrained Discrete Time Systems: Characterization and Implementation", was awarded the Eryl Cadwaladr Davies Prize as the best Ph.D. thesis in the Electrical and Electronic Engineering Department of Imperial College London in 2005.

Dr. Raković held a number of (research/academic/visiting) positions at the top universities worldwide, including Imperial College London, ETH Zürich, Oxford University, the University of Maryland at College Park and Supélec at Gif–sur–Yvette. Dr. Raković is presently a visiting scholar, research fellow with the Centre for Space Research of the University of Texas at Austin.

Dr. Raković has authored/co–authored more than 85 papers published in leading international journals and proceedings of key conferences in the fields of control theory and engineering. Many of his articles have become highly cited over a relatively short time span.

Dr. Raković's main academic interests and contributions lie within the areas of synthesis of control systems, analysis of dynamical systems and decision making under constraints and uncertainty. In particular, his contributions to robust model predictive control (more precisely, the development of the several generations of the so–called tube model predictive control design methods) and set invariance (namely, the development of the theoretical frameworks and algorithmic methods for the characterization and computation of invariant sets as well as set–induced Lyapunov functions) have reached a high level of international recognition.

B. William S. Levine

Dr. Levine is Research Professor of Electrical and Computer Engineering at the University of Maryland. He specializes in systems, control and estimation. He is engaged in research on control systems design, on the analysis and synthesis of biomechanical controls, and on signal processing issues related to controls. Current specific application projects include understanding the regulation of posture by humans, and reducing the time required to compute model predictive control signals.

Dr. Levine received his S.B., S.M. and Ph.D. degrees from MIT in Electrical Engineering. His Ph.D. thesis was entitled "Optimal Output Feedback Controllers for Linear Systems". He is a Fellow of the IEEE, a Distinguished Member of the IEEE Control Systems Society and a recipient of the IEEE's 3rd Millennium Medal. He is a past president of the IEEE Control Systems Society and of the American Automatic Control Council.

He has been an associate editor of the IEEE Transactions on Automatic Control, an editor of Automatica for Rapid Publications, program and, later, general chairman of the IEEE Conference on Decision and Control (CDC), and a member of the board of governors of the IEEE Control Systems Society. He is currently Chairman of the SIAM SIAG on Control Theory and its Applications.

Dr. Levine has many publications dealing with control and estimation theory and their application to practical problems. He is the co-author of the book Using MATLAB to Analyze and Design Control Systems (second edition 1995) published by Benjamin/Cummings. He is the editor of The Control Handbook (second edition 2010) published by CRC Press. He is the co-editor of the Handbook of Networked and Embedded Control Systems published by Birkhäuser. He is also the editor of a series of books on control engineering for Birkhäuser.

One of his papers, "Discrete Time Point Processes in Urban Traffic Queue Estimation" (co–authored with J.S. Baras and T.S. Lin) received a biannual award as the best paper in the IEEE Transactions on Automatic Control.

C. Behçet Açıkmeşe

Behçet Açıkmeşe received his Ph.D. in Aerospace Engineering in 2002 from Purdue University. He was a senior technologist at NASA Jet Propulsion Laboratory (JPL) from 2003-2012, and a lecturer in GALCIT at Caltech. At JPL, Dr. Açıkmeşe developed Guidance, Navigation, and Control (GN&C) algorithms for planetary landing, formation flying spacecraft, and asteroid and comet sample return missions. He was the developer of the "flyaway" GN&C algorithms in Mars Science Laboratory, which successfully landed on Mars in August 2012. His convex optimization based planetary landing algorithm is currently being flight–tested for future Mars missions. He later joined the Department of Aerospace Engineering and Engineering Mechanics at the University of Texas at Austin in 2012, where he is currently an Assistant Professor until December 31, 2015. Behçet Açıkmeşe will join the Department of Aeronautics and Astronautics at University of Washington as an Associate Professor from January 01, 2016. Dr. Açıkmeşe is an Associate Fellow of AIAA and a Senior Member of IEEE.

D. Ilya V. Kolmanovsky

Ilya V. Kolmanovsky has received his M.S. and Ph.D. degrees in aerospace engineering, and the M.A. degree in mathematics from the University of Michigan, Ann Arbor, in 1993, 1995, and 1995, respectively. He is presently a professor in the department of aerospace engineering at the University of Michigan, with research interests in control theory for systems with state and control constraints, and in control applications to aerospace and automotive systems. Dr. Kolmanovsky has previously been with Ford Research and Advanced Engineering in Dearborn, Michigan, for close to 15 years. He is a Fellow of IEEE, a past recipient of the Donald P. Eckman Award of American Automatic Control Council, and of IEEE Transactions on Control Systems Technology Outstanding Paper Award

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